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Application of Hot-Wire Anemometry and Digital Techniques to Measurements in a Turbulent Helium Jet

JOHN WAY* AND PAUL A. LIBBY†

Department of the Aerospace and Mechanical Engineering Sciences, University of California, San Diego, La Jolla, Calif.

A two-sensor "hot-wire" probe consisting of a wire and a film is used in connection with an absolute calibration procedure and with digital recording to provide time resolved data on one velocity component $u(t)$ and mass fraction of helium $c(t)$ in a turbulent helium jet discharging at low speed into quiescent air. A careful assessment of probe and system accuracy is provided. The principle new results relate to statistical quantities, i.e., intensities, cross-correlations and spectra, of velocity and concentration on the axis of a turbulent helium jet but some secondary results from other jets are presented.

I. Introduction

THERE are many turbulent flows of practical interest involving significant density fluctuations, e.g., in the wake of bodies in hypersonic flight, in propulsion units employing supersonic combustion, and in boundary layers with external streams having supersonic or hypersonic velocities. In many of these flows measurements of mean quantities such as velocity, temperature, and composition have been made and have been incorporated in methods of analysis. However, few measurements of fluctuating quantities in such flows have been made because of the experimental difficulties involved. In fact, Laufer[1] recently stated "It is somewhat disconcerting, for instance, that since the work of Kistler . . . (1959) . . . no experiments have been reported on turbulent fluctuations in a compressible flowfield above Mach four."

In addition to applied interest in turbulent flows with significant density fluctuations we note that the preponderance of research on the theory of turbulence relates to constant density flows. The necessity of experimental data to support and to guide theoretical developments in the case of turbulent flows with variable density clearly supports an effort to measure fluctuating quantities in such turbulent flows

In view of this situation we have undertaken a program of experimental investigation with the objective of measuring fluctuating quantities in fundamental flows involving significant density fluctuations and under conditions wherein high accuracy could perhaps be realized. These considerations suggest the study of simple, low-speed isothermal flows involving the mixing of two gases of widely different molecular weights. In such flows the density fluctuations are associated with concentration fluctuations. We thereby significantly reduce the problem of frequency response associated with more directly applicable, high-speed flows in which density fluctuations are generally due to temperature as well as perhaps to concentration fluctuations.

Our purpose here is to report the development of a system based on hot-wire anemometry and digital techniques to measure with time-resolution one velocity component and helium concentration and to present the results of its initial application to the measurement of fluctuating quantities on the axis of a low-speed helium jet discharging into quiescent air. In the course of assessment of system accuracy we provide some new data on other jet flows.

Our approach of using the signals from several hot-wires in order to obtain multiple data, e.g., in our case one velocity component and concentration of helium, is not novel. Corrsin[2] put forth and analyzed the possibilities of doing so in

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Index Categories: Jets, Wakes, and Viscid-Inviscid Flow Interactions; Research Facilities and Instrumentation.

* Assistant Research Engineer; presently at the Department of Mechanical and Aerospace Engineering, Illinois Institute of Technology, Chicago, Ill. Member AIAA.

† Professor of Aerospace Engineering. Fellow AIAA.

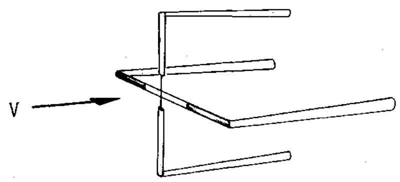

Fig. 1 Schematic representation of the probe.

1949. However, there appears to have been no serious effort to exploit these possibilities until the last few years. There have been several recent but unsuccessful attempts along lines similar to ours (cf. Tombach[3]). Tombach[3] abandoned this approach when he found that his wires suffered a history effect from exposure to high concentrations of helium. We have not encountered this difficulty because our measurements have never involved concentrations of helium greater than 30–40%.

II. Calibration, Data Collection, and Data Reduction

We have previously reported in Ref. 4 that a probe which provides satisfactory separation of the velocity component in the direction of the mainstream, denoted by u, and of the mass fraction of helium, denoted by c, is as shown in Fig. 1. It consists of a relatively cool wire orthogonal to, slightly upstream of, and generally within the thermal field of, a relatively hot film sensor. Both the wire and the film are orthogonal to the mean flow. Throughout the present discussion the subscript w will denote quantities related to the wire and the subscript f those related to the film. With this probe the thermal field of the film alters the electrical power required to maintain the wire at a constant temperature, the alteration depending on the velocity of the flow and the concentration of helium. On the other hand the wire does not significantly effect the film because it is at a relatively low temperature and has a diameter small compared to the length of the film.

For the present experimental results the wire consists of a 0.0001-in.-diam platinum element approximately 0.015 in. long. The hot film is 0.001 in. in diameter with an active length of quartz coated platinum 0.010 in. long. The surface-to-surface spacing in the main stream direction between the two elements is approximately 0.001 in. The wire and film are operated at 125°C and 300°C overheat, respectively.

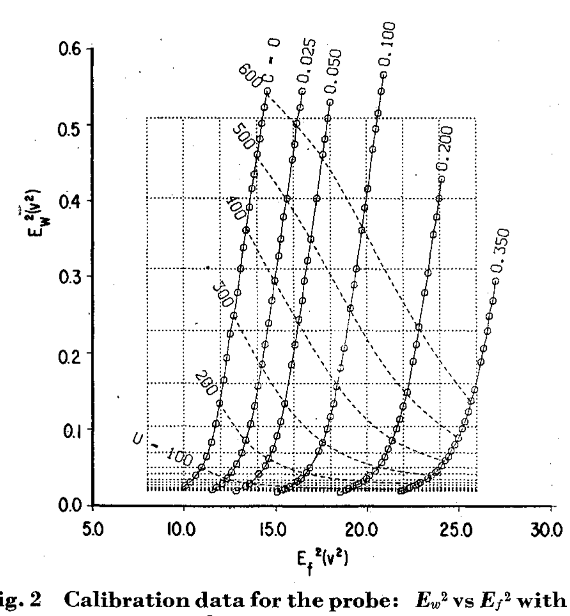

Fig. 2 Calibration data for the probe: E_w^2 vs E_f^2 with contours of constant u, constant c.

Our previous work (cf. Ref. 4) had shown that the square of the voltage drop across a single platinum wire required to maintain the wire at a constant temperature closely follows a King's law, $E_m^2 = A_m(c) + B_m(c)(u)^{1/2}$, where we use the subscript m to denote "master wire." Accordingly, two such wires are used in calibrating the two-sensor, data probe and permit us to extrapolate the calibration of each wire to velocities lower than could be accurately measured with our manometer. All sensors, the three wires and the hot film, are operated in the constant temperature mode; the data probe is powered by Thermo Systems Incorporated Model 1054A bridge circuits while the master wires are powered by DISA circuits.

Calibration is carried out at six values of the concentration of helium, $c = 0$, 0.025, 0.05, 0.10, 0.20, and 0.35 and over the velocity range from 25–1000 cm/sec. Previous experience had indicated that these ranges would bracket the velocities and concentrations on the axis of the helium jet at our measuring stations, namely at $x/d = 15$ and 20, where x is the distance from the orifice and d is the diameter of the orifice.

Standard gas storage bottles are charged with the desired mixtures of helium and air and are allowed to stand for periods of a week or more to assure homogeneity. The calibration is performed at the exit plane of a 0.75 × 1 in. jet. The pressure drop across the contraction section is measured with a precision manometer and is used with each gas mixture to establish a series of known velocities which serve to calibrate the master wires. Except for these pressure data and the corresponding voltage data required to calibrate the master wires, all other calibration data are voltage signals from the four sensors and are recorded directly on magnetic tape. The data collection system consists of SDS equipment: sample and hold amplifiers, an electronic multiplexer, and an analog-to-digital converter. Data are stored using a Kennedy digital tape recorder. Calibration and turbulent data are recorded at the rate of 2085 samples on each channel per second, each sample being twelve bits of information.

After the pressure and master wire data are collected for a given mixture, the output from the four sensors are recorded as the velocity in the calibration jet is changed over the range indicated above. This procedure is followed for each of the six gas mixtures.

The first step in data reduction involves establishing the calibration map of the data probe. For each gas mixture the calibration constants of the master wires are determined and then the triplets, E_w^2, E_f^2, u, computed. The essential results of this procedure are shown in Fig. 2, which is the calibration map for the data probe, namely, the loci of constant composition and constant velocity in the E_w^2, E_f^2 plane.

We have discussed previously[4] that these interfering probes we have employed in the present research fail to give useful signals at low velocities and high concentrations of helium if the influence of the film on the wire is excessive. On the other hand if there is too little such influence, then at high velocities and low concentrations it is not possible to determine unambiguously velocity and concentration. We see from Fig. 2 that for the range covered by our calibration the present probe has somewhat too much influence of the film on the wire. However, we find a posteriori that voltage pairs obtained in the turbulent flow on which we report in the present work seldom appear in the lower right corner of the calibration map corresponding to low velocities and high concentrations. Accordingly for our purposes the present probe is satisfactory.

From Fig. 2 and the data used to construct it we can establish two surfaces $u = u(E_w^2, E_f^2)$, $c = c(E_w^2, E_f^2)$; from this point of view inverting voltage data to provide u, c data requires two, two-dimensional interpolation schemes. Accordingly, we divide the E_w^2, E_f^2 domain enclosing all voltage pairs obtained from turbulent flow into a Cartesian grid with unequal mesh sizes. By appropriate spline fitting of the calibration data of Fig. 2 the values of u and c at the intersection points of the grid are found and may be used to establish within each mesh a local, quadratic description of each surface, $u = u(E_w^2, E_f^2)$ and $c = c(E_w^2, E_f^2)$.

We have found that on a CDC 3600 six seconds is required to invert 2048 voltage pairs corresponding to approximately one second of turbulent flow data to produce the same number of u, c pairs and several statistical quantities and that 160 mesh points, as shown in Fig. 4, provide adequate resolution. We remark that the close spacing of the grid lines for low values of E_w^2 is observed in Fig. 2.

Measurements have been made in a turbulent isothermal flow from a circular jet, 1 in. in diameter in the middle of a horizontal disk, 5 ft in diameter. The jet discharges vertically into quiescent air and issues from an axisymmetric contraction section attached to a settling chamber of 3.6 in. diam with stilling screens. A thermometer in the settling chamber monitors the jet temperature which for all tests reported varied less than 1°C from ambient room temperature.

Three concentrations of helium in the initial jet flow, denoted c_o, are considered: $c_o = 0, 0.10, 1.0$. The first corresponds, of course, to an air jet and is used principally to provide data permitting assessment of the accuracy of the system. The tests with $c_o = 0.10$, termed the 10% helium jet, correspond to employing helium as a tracer and thus to a jet with negligible density effects. The results from these tests can be compared with existing data from slightly heated air jets and thus can again be used principally for assessment of accuracy although some new data are in fact obtained. The principle new results of our program are those obtained when the jet is pure helium, $c_o = 1$.

In addition to these jet flows, two tests providing further assessment of system accuracy are considered; these involve placing coaxially with the circular orifice a circular tube of 1.5 in. in diam and 16 in. in length and collecting data corresponding to turbulent pipe flow in air ($c = 0$) and in a 10% helium-air mixture of fixed composition.

The voltage pairs collected under conditions wherein the composition is known and constant with time, i.e., the air jet and the pipe flows, can be treated in two distinct ways. By employing the data reduction procedure described above such pairs yield a time series of velocity and apparent composition. Comparison of the apparent and known compositions provides an assessment of accuracy. The second way to treat these voltage pairs is to force the composition to be its known value and to obtain two time series of velocity. Comparison of all three times series of velocity provides another assessment of accuracy.

We remark now on the exit velocities of the jets denoted by u_o. We adjust these velocities so as to reflect the maximum sampling rate available to us. With that rate equal to 2085/sec per channel, we desired to adjust the flow velocity so as to keep negligible the contribution of frequencies over 1000 cps. Preliminary tests had indicated that for the helium jet ($c_o = 1$) a velocity of 1500 cm/sec would yield power spectra satisfying this condition. The smaller diameter in the air jet at the same near field station requires u_o to be reduced and we thus set u_o for the air and 10% helium jets at about 600 cm/sec. For the pipe tests mean velocities of 200–400 cm/sec, representative of those obtained in the jet flows at $(x/d) = 15$, are set.

These considerations lead to actual test values of $u_o = 610$ cm/sec for $c_o = 0, 0.1$ and to $u_o = 1525$ cm/sec for $c_o = 1.0$. The Reynolds numbers based on orifice diameter and exhaust conditions for these three cases are 10,250, 7170, and 3290, respectively.

For a particular test, voltage pairs are collected for nearly one minute in the format of approximately 220 records with 512 samples in each. Averaging periods are taken to correspond to four records, almost one second; the selection of this period has the effect of removing from our statistical analysis low frequency, spurious excursions of the jet. We note, however, that in most cases the differences in relative intensities are changed by less than 5% by averaging over all records.

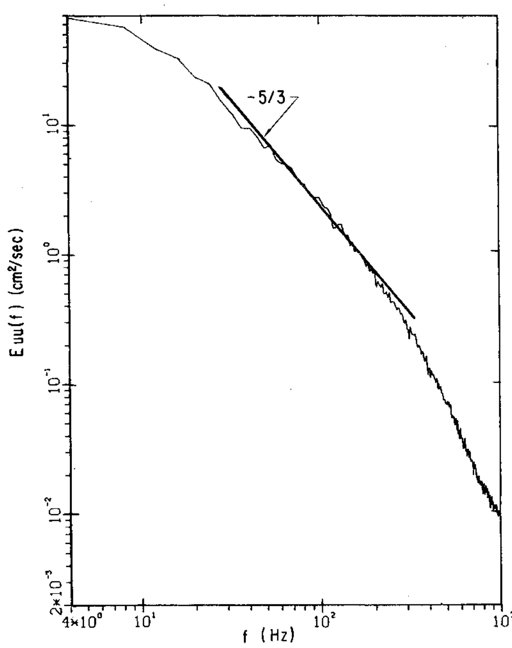

Fig. 3 Typical power spectra: Air jet, $x/d = 15$.

As part of our data reduction process we have used the various time series of u and c in conjunction with the techniques of the discrete Fourier transform to obtain one-dimensional power spectral densities, denoted $E(f)$. To indicate our procedures, suppose we have a time series z_k, $k = 0, 1, 2, \ldots, N - 1$, of real values corresponding to the values of z at equal time intervals Δ_t. Then the rth coefficient of the discrete Fourier transform is defined to be

$$Z_r = N^{-1} \sum_{k=0}^{N-1} z_k \exp(-i2\pi rk/N), \; r = 0, 1, 2, \ldots \tfrac{1}{2}N - 1$$

where Z_r corresponds to the frequency $f = r/N\Delta_t = r\Delta_f$. The one-dimensional power spectral density related to Z_r is

$$E_{zzr} = |Z_r|^2 N\Delta_t$$

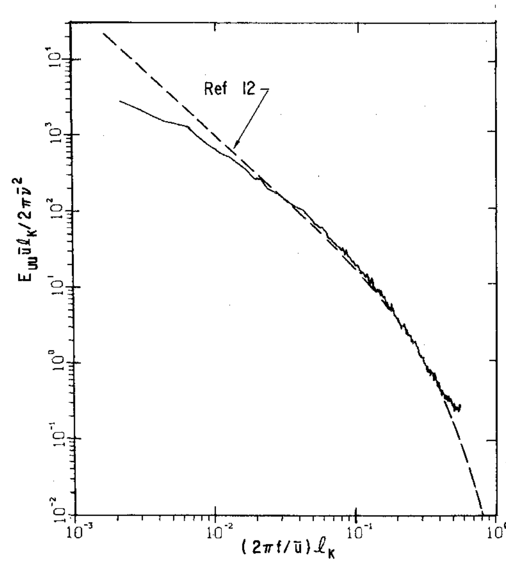

Fig. 4 Universal velocity spectrum: helium jet, $(x/d) = 15$.

Note that in accordance with the above definitions

$$\overline{z^2} \simeq N^{-1} \sum_{k=0}^{N-1} z_k^2 = 2(N\Delta_t)^{-1} \sum_{r=0}^{\frac{1}{2}N-1} E_{zzr}$$

In our data reduction, we treat the fluctuations in each record associated with one second averaging as a time series. With a data collection rate of 2085/sec, $\Delta_f = (N\Delta_t)^{-1} \simeq$ 4Hz. To obtain $E_{zz}(f)$ we average the related E_{zzr} over all records available.

Similarly, suppose we wish to obtain cross spectra and cross correlations. We consider two time series y_k, z_k, $k = 0, \ldots. N-1$ of equal length and equal time interval, Δ_t. Then the coefficients of the discrete Fourier transform of each series, Y_r and Z_r, $r = 0, 1, 2, \frac{1}{2}N - 1$, yield the power spectral density at frequency $f = r\Delta_f$

$$E_{yzr} = |Y_r Z_r^*| N\Delta_t$$

where Z_r^* is the complex conjugate of Z_r. As before we compute E_{yzr} for each record and average over all records available. Furthermore, to compute the cross correlation of the two series we employ the inverse discrete Fourier cosine transform of the series E_{yzr} to find

$$\hat{E}_{yzl} = \sum_{r=0}^{\frac{1}{2}N-1} E_{yzr} \cos(2\pi rl/N),\ l = 0, 1, \ldots, \tfrac{1}{2}N - 1$$

and then compute the cross-correlation coefficient

$$R_{yzl} = \hat{E}_{yzl} \Bigg/ \left(\sum_{k=0}^{N-1} y_k^2\right)^{1/2} \left(\sum_{k=0}^{N-1} z_k^2\right)^{1/2}$$

where l is the number of multiples of Δ_t between the terms in the two series being correlated. In general $R_{yzo} \neq 1$ but does correspond to the correlation coefficient of the usual one-point, one-time correlation of the time series y and z, e.g., for the correlation of velocity and concentration fluctuations.

We have computed from our spectral results the universal constants α and β associated with the inertial subrange of the spectra of the fluctuations of velocity and concentration respectively, not only to add credibility to our data but also to supply additional measurements for these constants whose accepted values presently appears to be in doubt.[5,6]

Central to the calculation of these constants is the estimation of the mean rate of viscous dissipation ϵ and of the mean rate of diffusive dissipation χ. We follow standard practice[6] and thus compute first

$$\epsilon = (2\pi)^2(15\nu\bar{u}^2) \int_0^\infty f^2 E_{uu}(f)df$$

$$\chi = (2\pi)^2(6D/\bar{u}^2) \int_0^\infty f^2 E_{cc}(f)df$$

where

$$\int_0^\infty E_{uu}df = \overline{u'^2} \text{ and } \int_0^\infty E_{cc}df = \overline{c'^2}$$

We indicate integrals here because we actually fair our discrete spectral values and perform a graphical integration taking into account an estimate of the small contribution to the integrals from the high frequency end, i.e., beyond the frequency at which the signal-to-noise ratio approaches unity. We then select values of $E_{uu}(f)$ and $E_{cc}(f)$ and their associated values of f in the inertial subrange of f and compute

$$\alpha = (2\pi/\bar{u}\epsilon)^{2/3} f^{5/3} E_{uu}(f)$$

$$\beta = (\epsilon/\chi)^{1/3}(2\pi/\bar{u})^{2/3} f^{5/3} E_{cc}(f)$$

III. Assessment of Accuracy

Assessment of system accuracy is carried out at several levels. The first relates to establishing the accuracy with which the voltage pairs corresponding to calibration data when fed back into the data reduction system actually reproduce the calibration pairs of u and c. We have carried out this step in several ways and conclude that the data reduction scheme is accurate within an error of less than 1% in regions of the E_w^2, E_f^2 plane involving most of the voltage pairs obtained from turbulent flow. In extreme regions of the map, in particular in the lower right hand corner corresponding to high concentrations of helium and low velocities, less than 50 cm/sec, regions which correspond to infrequently occurring voltage pairs, the accuracy degrades due to digitizing errors and to lack of sensitivity of the wire. No significant errors result therefrom.

The second level at which assessment of system accuracy is carried out relates to the self-consistency of the results for the flows with known, constant concentration. The distillation of this assessment is shown in Table 1. We consider first the self-consistency of the air jet results; note first that the apparent mean concentrations, which ideally should be zero, are less than 10^{-3} at both x/d stations. Next, note that the mean velocities indicated by $\bar{u}/u_o$ and the turbulent intensities indicated by $(\overline{u'^2})^{1/2}/\bar{u}$ as given by all three sets of velocity data agree within five percent with one another. However, the five percent discrepancy is not random, the velocity indicated by the film being consistently lower than that indicated by the wire. We have carefully investigated the reason therefore and have inferred that the film voltages incurred a spurious drift of less than 1%. This does not affect significantly the accuracy of results when u and c are determined or when the concentration is forced and the wire voltages are used to determine velocity but it does lead to the indicated errors when the film voltages are used to determine velocity. This conclusion is supported by consideration of Fig. 2 where

Table 1 Results of system assessment

c_o	x/d	$\bar{c}$	$\bar{c}/c_o$ Present results	$\bar{c}/c_o$ Previous results	$\overline{c'^2}^{1/2}/\bar{c}$ Present results	$\overline{c'^2}^{1/2}/\bar{c}$ Previous results	$\bar{u}/u_o$ Present results	$\bar{u}/u_o$ Previous results	$\overline{u'^2}^{1/2}/\bar{u}$ Present results	$\overline{u'^2}^{1/2}/\bar{u}$ Previous results	R_{cuo} Present results	R_{cuo} Previous results	$\hat{\tau}_{ou}$	α	l_K, cm	Notes
0.0	15	−0.0008					0.38	0.48[7]	0.25	0.20[8]			0.18	0.44	0.018	Probe
0.0	15						0.37	0.45[6]	0.22	0.33[3]						Film
0.0	15						0.39	0.41[9]	0.26					0.46		Wire
0.0	15							0.36[3]								
0.0	20	−0.0008					0.31	0.34[7]	0.24	0.24[7]			0.16	0.46	0.023	Probe
0.0	20						0.30	0.33[8]	0.22	0.21[8]						Film
0.0	20						0.31	0.32[9]	0.25	0.33[3]				0.47		Wire
0.0	20							0.26[3]								
0.0		−0.0019					197[a]		0.071	0.03[10]				0.39		Pipe-probe
0.0							201[a]		0.074					0.40		Pipe-wire
0.1	15	0.023	0.23	0.30[2]	0.24	0.13–0.17[2]	0.34	0.40[8]	0.24	0.17–0.22[8]	0.42	0.19–0.29[8]	0.19	0.48	0.022	
0.1	20	0.017	0.17	0.22[2]	0.24	0.13–0.16[2]	0.27	0.30[8]	0.24	0.18–0.22[8]	0.38	0.15–0.29[8]	0.15	0.42	0.027	
0.1		0.099					387[a]		0.070	0.03[10]				0.39		Pipe-probe
0.1							389[a]		0.072					0.41		Pipe-wire

[a] $\bar{u}$ (cm/sec).

it may be seen that a small error in E_f^2 with the concentration constrained will lead to a significant error in velocity. This drift of film output accounts for all the errors in apparent concentration being negative. Accordingly, in subsequent entries in Table 1 we show only combined results and those from the wire voltages with constrained concentration. This result is discussed in detail to give an indication of the sensitivity of results to small errors and of the need for extreme care in performing the experiments.

The 10% helium jet and the pipe flow results given in Table 1, considered from the point of view of self-consistency, follow closely those for the air jet, i.e., the probe gives an apparent mean concentration within roughly 0.001 of the known value and the velocities indicated by the probe and by the wire with concentration constrained are within 2% of one another.

We show in Fig. 3 a typical power spectrum $E_{uu}(f)$ to indicate that even for the relatively low Reynolds numbers of the flows studied here a range of close to $(-\frac{5}{3})$ decay can be identified so that meaningful estimates of the universal constants α and β of the inertial subrange can be made. We thus show in Table 1 the several values of α obtained from these assessment tests and some of the associated Kolmogoroff lengths, l_K. In connection with the identification of an inertial subrange we note that for all the jet flows discussed here the Reynolds number $R_\lambda \equiv \overline{u'^2}(15/\epsilon\nu)^{1/2}$ is found to be somewhat greater than 200.

A review of the data in Table 1 from the point of view of self-consistency indicates that the mean apparent concentrations are in the absolute sense within 0.001 of the true concentration, the mean velocity and relative intensity data given by the combined sensors and by the wire alone agree within 2% and that the α values are all in good agreement in terms of the accuracy bands usually associated with such numbers.

We now turn our attention to the third level of assessment, that concerned with comparison of the results shown in Table 1 with relevant, previously published data. As an initial remark we observe that such comparisons performed for jet flows are generally compromised by differences in Reynolds numbers and in details of the orifices and of orifice mounting so that precise agreement cannot be expected even if both experiments are free of error.

We compare our air jet results with the recent data of Wygnanski and Fiedler,[7] with the frequently quoted data of Corrsin and Uberoi,[8] with the early, less frequently quoted, data of Hinze and van der Hegge Zijnen,[9] and finally with the air jet data which are given by Tombach[3] but which are secondary to his purposes. The Reynolds numbers based on orifice flow for these four references are $100(10)^3$, $34\text{–}60(10)^3$, $67(10)^3$, and $15(10)^3$ respectively, generally considerably greater than that of our flow. For comparison with Refs. 8 and 9 we have assumed that their velocity data with the temperature of the air at the orifice 15°C and 30°C above ambient may be considered to correspond to an unheated jet. Comparison of the present results with those of Refs. 3, and 7–9 as given in Table 1 is considered to be quite satisfactory. In view of the differences in the measurements presented previously. To be more explicit consider one typical quantity, the relative intensity $(\overline{u'^2})^{1/2}/\bar{u}$ at $x/d = 20$. For the air jet from the probe and the wire we find 0.24 and 0.25, respectively, whereas previous values range from 0.21–0.33.

We next turn to the comparison of our time autocorrelation data with those of Wygnanski and Fiedler[7] for the air jet at $x/d = 20$. We compute $R_{uu\ell}$ as a function of the normalized time delay $\hat{\tau}_u \equiv \bar{u}\tau/x$ where $\tau = l\Delta_t$ and present our results in terms of the value of $\hat{\tau}_u$ for which R_{uu} goes to zero. Denote this as $\hat{\tau}_{uo}$ which we show in Table 1. In Ref. 7 only results for $R_{uu\ell} > 0$ are given and then only for $(x/d) = 30, 40,$ ... However shown there is an insensitivity of R_{uu} with (x/d) when presented in terms of $\hat{\tau}$ and a value of $\hat{\tau} = \hat{\tau}_o =$ 0.18 for $R_{uu} = 0$. Thus we consider the agreement with our results which give $\hat{\tau}_o = 0.16$ to be satisfactory. We note that one record, i.e., the period used to compute our statistical data, corresponds to $\hat{\tau} = 0.93$ which is sufficiently greater than $\hat{\tau}_o$ to give confidence in the adequacy of the data sample. Furthermore the value of $\hat{\tau}$ corresponding to one record is greater than 0.93 for all other jet flows.

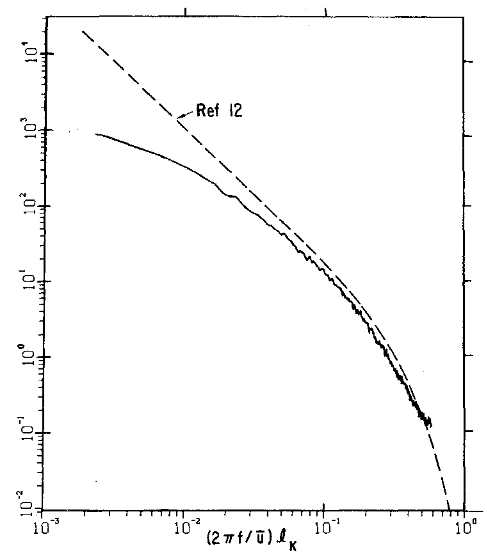

Fig. 5 Normalized concentration spectrum: helium jet, $(x/d) = 15$ (see text for ordinate).

We consider now the comparison of our pipe results with those of Laufer[10] at a Reynolds number of roughly 10^2 times greater. We note that our relative intensity is twice as great as that of Laufer. This discrepancy is reduced to about 35% by comparing $(\overline{u'^2}/u^*)$ where u^* is the shearing velocity, However, we conclude that the inlet conditions to our pipe. corresponding as they do to an abrupt expansion from 1 in. diam–1.5 in. diam, and the shortness of our pipe result in our observed higher intensity. Thus our pipe data should be considered principally on the basis of self-consistency although the value of α is comparable with other data.

We compare next the 10% helium jet results with the heated jet data of Ref. 8. In doing so, we take their data for a jet temperature of 170°C above ambient because this most closely approximates in terms of the density ratio, orifice to ambient, the value obtained with a 10% mixture of helium and air. In connection with these results we note that the Schmidt number at our two measuring stations is roughly 0.24, whereas the transport parameter relevant for heated jets is the Prandtl number, i.e., 0.7 for air. Again considering the differences in Reynolds number and the imprecision of the equivalence of using helium and temperature as tracing elements the agreement shown in Table 1 is considered quite satisfactory. In particular consider again the relative velocity intensity at $x/d = 20$; we find 0.24 compared to the two values 0.18 and 0.22 given in Ref. 8.

We remark next on the comparison of the constant α from the present results with previous values and note that all our values given in Table 1 are between 0.39 and 0.50 in good accord with "the most recent determinations" of this "constant" (cf., e.g., Ref. 11). In this connection we note that Gibson et al.[6] have inter alia provided an assessment of the various measurements of α since they obtained the high value of $\alpha = 0.69$ in the air boundary layer over the open ocean.

With respect to the value of β inferred from the 10% helium jet results: at $x/d = 15$ and 20 we find $\beta \simeq 0.25$; this should be compared with the value of 0.35 quoted by Panofsky[11] as having "considerable agreement" associated with it; with the first measurement thereof, namely 0.33–0.44, given by Gibson and Schwarz[12]; and with 0.31 obtained by Grant et al.[13] In connection with the lower value we find for β we note that Gibson et al.[6] have recently obtained $\beta = 1.17$ from measurements of temperature in the air boundary layer over the ocean, have offered two explanations of their dramatically high value, and have stated that "further measurements of β both at low and high Reynolds numbers are needed. . . ." Thus the present value may be of interest.

IV. Helium Jet Results

We show in Table 2 the principle results obtained from the helium jet; shown there are the mean values of velocity and concentration, the relative intensities of the velocity and concentration fluctuations, the cross-correlation coefficients of the velocity and concentration fluctuations, and the relative intensities of density fluctuations. A limited number of these data can be compared with previous results. In particular, there are available the early measurements of mean quantities $\bar{c}$, and $\bar{u}/u_o$ of Keagy and Weller[14] obtained by sampling probes and the recent measurements due to Tombach[3] of $\bar{u}/u_o$ and $(\overline{u'^2})^{1/2}/\bar{u}$ by means of a time-of-flight probe. The previous remarks concerning unequivocal comparison of the present and previous data pertain here as well. We note, that our conditions at the jet exit appear to lead to more rapid decay of the jet than obtained by Keagy and Weller.[14]

Comparison of results for the pure helium jet with those of the 10% helium jet provide a direct indication of the effect of variable density if we consider of secondary importance the difference by a factor of two in the Reynolds numbers of the two flows. We list in Table 2 the relative density intensity, $(\overline{\rho'^2})^{1/2}/\bar{\rho}$, roughly 10%; this may be taken as a measure of the density effects in the jet flow under consideration. With respect to mean quantities, $\bar{u}/\bar{u}_o$ and $\bar{c}/c_o$ we see the well-known increase in mixing rate in the near field associated with jets of density less than ambient, an increase accounted for roughly by an effective jet diameter reduced by the square root of the density ratio.[15] Despite these lower relative mean values the relative intensities of velocity and concentration fluctuations in the helium jet are higher than in the 10% helium jet and the concentration and velocity are somewhat more correlated. Whether these results are general or are due to differences in the two jets aside from density effects cannot at present be determined.

We now turn our attention to spectral information from our helium jet results. First we list in Table 2 our values for $\hat{\tau}_o = \bar{u}\tau_o/x$ where τ_o is the time for zero autocorrelation since this provides a measure of an integral scale. We show both $\hat{\tau}_{ou}$ corresponding to the velocity autocorrelation and $\hat{\tau}_{oc}$ corresponding to the concentration autocorrelation since they are not identical. Comparison of $\hat{\tau}_{ou}$ with those from the air jet leads to the expected result that the integral scales are somewhat greater in the helium jet, roughly consistent with the greater radial extent of the helium jet.

The next question concerns the spectra of the fluctuations of velocity and concentration in the helium jet. There is no theory which provides a basis for treating such spectra and there is no indication from either theory or previous data as to whether density fluctuations of the degree we measure are spectrally significant. However, we adopt a heuristic point of view and treat the time series of velocity and concentrations in the same fashion as for the air and 10% helium jets, i.e., for flows of constant properties; in doing so we replace the kinematic viscosity by its value computed at the measured mean concentration. We thereby neglect the influence on viscous and diffusive dissipation of fluctuations in the transport properties. There results from this procedure values of the constant for the inertial subrange of the velocity spectra, $\alpha = 0.50$ and 0.46 at $(x/d) = 15$ and 20, respectively, and of the concentration spectra, $\beta = 0.27$ and 0.26 at $(x/d) = 15$ and 20 respectively. These values closely agree with the corresponding values for the air and 10% helium jets and lead to the conclusion that at least for the high frequency portion of the spectra the effect of density fluctuations of the magnitude present in the flows studied here is unimportant.‡ We show in Table 2 the Kolmogoroff lengths which result from this calculation and find them to be surprisingly the same as those for the other jet flows.

The above conclusion about the similarity of the spectra is supported by examination of the detailed spectra. We show in Fig. 4 the velocity spectra presented in universal form, i.e., as $(E_{uu}\bar{u}l_K/2\pi\bar{\nu}^2)$ vs $(2\pi f/\bar{u})l_K$ for the helium jet at $(x/d) = 15$ and compare them with the universal form which is provided by Gibson and Schwarz[12] and which is generally considered applicable to turbulent flows with uniform properties. It may be seen that for $(2\pi f/\bar{u})l_K \gtrsim 0.02$ the present data is in close agreement with the universal form.

With respect to the spectra of the concentration fluctuations we have carried out a calculation similar to that described above for the velocity fluctuations. However, in this case the universal spectra depend on Schmidt number (or generalized Prandtl number) which is 0.29 and 0.26 at $(x/d) = 15$ and 20, respectively, for the helium jet. Moreover, there appear to be no previous measurements of concentration spectra for this Schmidt number range even for flows with essentially constant properties. Therefore in Fig. 5 we give the concentration spectra for the helium jet at $(x/d) = 15$ in the form $E_{cc}(f)\bar{u}l_K{}^3\bar{\nu}/2\pi\chi$ vs $(2\pi f/\bar{u})l_K$; the spectrum for the 10% helium jet presented in the same fashion is almost indistinguishable for $(2\pi f/\bar{u})l_K \gtrsim 0.02$. We also show on Fig. 5 for comparison the universal velocity spectra. From these data we conclude that the high frequency end of the concentration spectra is also not influenced by the density fluctuations in our experiments and that for the Schmidt number in the range of 0.24–0.29 the concentration spectra in appropriate form are close to that of the velocity spectra.

We now turn our attention to several results of possible engineering interest. In one form of the equations describing the conservation of mean momentum and species in compressible turbulent flows there arise the double correlations $\overline{(\rho u)'u'}$, $\overline{(\rho u)'c'}$.[16] Generally, these are decomposed into other double correlations and into a triple correlation; the latter is always neglected and in some cases one of the double correlations is neglected on the basis of heuristic arguments. But with our time series of velocity and concentration we can compute the original double correlations and their component parts and thereby can provide data which may be useful in guiding the aforementioned arguments and the subsequent modeling of these terms.

‡ George K. Batchelor suggested to the second author in a private communication that this might be expected to be the case since the high wave number fluctuations are dominated by viscous effects.

Table 2 Results from helium jet

x/d	$\bar{c}$		$(\overline{c'^2})^{1/2}/\bar{c}$		$\bar{u}/u_o$		$(\overline{u'^2})^{1/2}/\bar{u}$		R_{cuo}	$\hat{\tau}_{ou}$	$\hat{\tau}_{oc}$	l_K,cm	$(\overline{\rho'^2})^{1/2}/\bar{\rho}$
	Present results	Previous results	Present results	Previous results	Present results	Previous results	Present results	Previous results					
15	0.083	0.13[14]	0.33		0.16	0.25[14]	0.28	0.42[3]	0.54	0.28	0.35	0.022	0.12
15						0.15[3]							
20	0.053	0.10[14]	0.37		0.13	0.15[14]	0.30	0.45[3]	0.56	0.17	0.17	0.024	0.10

We show below two examples of these results obtained from the data collected at $x/d = 15$ in the helium jet:

$$\overline{(\rho u)'u'}/D_u = \bar{\rho}\overline{u'^2}/D_u + \bar{u}\overline{\rho'u'}/D_u + \overline{\rho'u'^2}/D_u$$
$$0.920 \qquad 1.200 \qquad -0.279 \qquad -0.002$$

where

$$D_u \equiv [\overline{(\rho u)'^2}\ \overline{u'^2}]^{1/2}$$

and

$$\overline{(\rho u)'c'}/D_c = \bar{\rho}\overline{u'c'}/D_c + \bar{u}\overline{\rho'c'}/D_c + \overline{\rho'u'c}/D_c$$
$$0.216 \qquad 0.686 \qquad -0.489 \qquad 0.020$$

where

$$D_c \equiv [\overline{(\rho u)'^2}\overline{c'^2}]^{1/2}$$

We see that for our particular case the neglect of one of the second order correlations compared to another may be questionable. There can be carried out similar examination of the component parts of other double correlations such as arise in other methods for decomposing the flow variables into mean and fluctuating parts.

Along similar lines we show how our data permit a quantitative assessment of two methods of averaging. Favre (cf., e.g., Ref. 17) has shown that the equations for conservation of mean quantities in turbulent compressible, more generally variable density, flows take on forms closely related to their counterparts in flows of constant properties provided the velocities and certain other dynamic variables are mass averaged. This method has evoked considerable interest (cf. Ref. 1) and would appear to afford distinct advantages for theoretical analyses. Therefore, we consider the differences between mass averaged and conventional averaged u and c; following Favre, we let $\tilde{u} \equiv \overline{\rho u}/\bar{\rho}$, $\tilde{c} \equiv \overline{\rho c}/\bar{\rho}$. But it is readily shown that

$$\tilde{u}/\bar{u} = 1 + \overline{\rho'u'}/\bar{\rho}\bar{u}$$

$$\tilde{c}/\bar{c} = 1 + \overline{\rho'c'}/\bar{\rho}\bar{c}$$

and from our data we can compute the correlation coefficients on the right hand sides. Typical results are given by the helium jet at $(x/d) = 15$, namely $(\overline{\rho'u'}/\bar{\rho}\bar{u}) = -0.020$ and $(\overline{\rho'c'}/\bar{\rho}\bar{c}) = -0.041$. We thus see that for the present experiments the differences between mass averaged and conventional averaged velocity and concentration are small.

Finally, it might be asked whether the probe used in the experiments reported here can be employed along with calibration and rms voltmeter to obtain mean velocity and mean concentration. To give an indication of the answer to this question we have computed for the helium jet results the $\overline{E_w{}^2}$ and $\overline{E_f{}^2}$ and from this voltage pair have computed apparent mean values of u and c. We find that within 10% the answer to the above question is affirmative; e.g., at $x/d = 15$ the apparent mean velocity and mean concentration computed from the mean squared voltages are 258 cm/sec and 0.078, respectively, compared to 244 cm/sec and 0.083 obtained from the full data recording and data reduction system.

V. Concluding Remarks

On the basis of the above results we conclude that our probe combined with digital recording and analysis provides a means for making measurements of fluctuations of concentration and of streamwise velocity in turbulent flows of helium and air. A requirement of high accuracy in the results appears to necessitate great care in calibration, testing and data reduction.

Our initial results as to the effect of density fluctuations in turbulent flows must be greatly expanded by further application of our technique before a reasonable picture of these effects emerges. However, one result of interest from the present results is that the high wave number end of the velocity spectrum is unaltered by the density fluctuations associated with a relative density intensity $(\overline{\rho'^2})^{1/2}/\bar{\rho}$ of 10%.

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